Classical Black Holes Are Hot

Erik Curiel

Munich Center For Mathematical Philosophy
Ludwig-Maximilians-Universität

and

Black Hole Initiative
Harvard University

erik@strangebeautiful.com

The Current State of Play in Physics

focus has been on quantum properties and effects; nothing of thermodynamical interest in a purely classical treatment of black holes

My Take on the Matter

black holes considered classically have a rich thermodynamical theory; there is a deep relation between gravitation and thermodynamics already, without taking quantum mechanics into account

The Status of the Analogy between the Laws?

A purely formal analogy? Or are black holes in classical general relativity truly thermodynamical objects? Are the black-hole laws the laws of thermodynamics extended to cover black holes?

The Standard Argument: No!

classical black holes are perfect absorbers
$\Longrightarrow$ must have physical temperature absolute zero, by laws of black-body radiation or by efficiency arguments

(Wheeler, Penrose, Hawking, Geroch, Wald, Israel, Carter, Sorkin, Jacobson, ...)

a little more precisely...

Put a black hole immersed in thermal radiation in a reflecting box; the black hole will perfectly absorb all the radiation and emit none. Thus, temperature absolute zero.

Problem of physical dimensions

- physical dimension $T = \texttt{mass}$
- physical dimension $S =$ pure scalar

BUT

- physical dimension $k = \texttt{mass}^{-1}$
- physical dimension $A = \texttt{mass}^2$

Ergo

must invoke quantum mechanics—derivation of Hawking radiation, natural introduction of $\hbar$ (physical dimension $\mathtt{mass}^2$)—to justify claim that κ is a physical temperature and A a physical entropy

Standard Argument Misses Real Physics

A black hole immersed in thermal radiation in a box *will* emit energy: in-falling thermal radiation perturbs the black hole causing it to emit gravitation radiation, which physically couples with the thermal radiation.

classical black holes are
not perfect absorbers

Need to Emend Physical Concepts

emission of gravitational energy appropriate notion of *thermal coupling* for purely gravitational systems, not electromagnetic (or other quantum-field) energy

(also derivation of Hawking radiation not nearly so good an argument for attributing temperature as usually thought)

Some Counterfactual History

Imagine we knew only general relativity and classical thermodynamics—how would we decide?

1. Can κ be introduced in the same ways and does it play the same formal roles as T?
2. Do κ and A mediate couplings to ordinary thermodynamical systems in the same way as T and S?

This is just how classical thermodynamics was extended to cover black-body radiation at the end of the 19th Century.

Black Holes as Truly Thermodynamical Objects?

surface gravity and area "couple" with temperature and entropy of ordinary thermodynamical systems in appropriate way

$$\Downarrow$$

surface gravity and area ARE physical temperature and entropy

Reversible Carnot-Geroch Cycle

1. empty box "at infinity", one side a piston; "quasi-statically" fill box with fluid from heat bath at fixed temperature; when piston has withdrawn part of the way, seal box; box now has total energy depending on rest mass, temperature, entropy, and work done
2. "quasi-statically" lower box towards black hole, extracting work from its gravitational energy; an observer inside box sees nothing change; "at infinity" measured total energy has decreased by redshift factor (work done)

Reversible Carnot-Geroch Cycle, cont.

3. at fixed proper distance from black hole, hold box stationary
4. “quasi-statically” draw piston back more, lowering temperature of fluid, keeping entropy unchanged, lowering total energy by work done; value of final temperature fixed so that change in total entropy vanishes during entire cycle

Reversible Carnot-Geroch Cycle, cont.

5. "quasi-statically" eject fluid; fluid falls into black hole delivering mass-energy (as heat), and positive entropy; piston returns to initial state

6. pull box back "to infinity" (takes no work), returning to its initial state

Assume:

1. conservation of energy (Generalized First Law)
2. ordinary entropy and entropy attributed to black hole are additive
3. the process can consistently be made isentropic

Then:

κ plays role of integrating factor for heat dumped in black hole, so κ is physically a temperature, with change in A as the integrated factor, so A is entropy; and there exists a universal constant α with physical dimension $\mathtt{mass}^2$ such that

1. $T_{\text{BH}} = 2\pi\alpha\kappa$
2. $S_{\text{BH}} = \dfrac{A}{4\alpha}$

so physical dimensions are correct

Quantity of Heat Transferred

(Δ total mass) - (Δ irreducible mass)

Conjecture (Clausius Postulate for Black Holes)

A transformation whose only final result is that the change in irreducible mass of black hole at a given surface gravity is less than the change in irreducible mass of one with a higher surface gravity is impossible.

Proposition (Kelvin Postulate for Black Holes)

A transformation whose only final result is to decrease the angular momentum and increase the irreducible mass of a black hole while leaving its surface gravity unchanged is impossible.

Derivation of Universal Constant

1. T_{BH} and κ are both integrating factors for the quantity of heat dumped into the black hole, turning it into the perfect differential $\text{d}A$ (from First Law)

2. therefore $\frac{T_{\text{BH}}}{\kappa} = \psi(A)$, for some function ψ

3. thus $\psi(A)\text{d}A = \text{d}S_{\text{b}}$ (also from First Law)

4. so $A = \theta(S_{\text{b}})$ (for some function θ)

Derivation of Universal Constant, cont.

5. now, consider 2 black holes, very far apart, run Carnot-Geroch cycles on each separately; both total area and total entropy dumped in are additive
6. thus $\theta_1(S_{b1}) + \theta_2(S_{b2}) = \theta_{12}(S_{b12}) = \theta_{12}(S_{b1} + S_{b2})$
7. and so $\frac{\mathrm{d}\theta_1}{\mathrm{d}S_{b1}} = \frac{\mathrm{d}\theta_2}{\mathrm{d}S_{b2}}$
8. since parameters of the 2 black holes are arbitrary, there must exist a universal constant α such that $\frac{\mathrm{d}\theta}{\mathrm{d}S_b} = \frac{\mathrm{d}A}{\mathrm{d}S_b} = \frac{1}{\alpha}$
9. it follows by construction α has proper physical dimension: $T_{\text{BH}} = 8\pi\alpha\kappa$ and $S_{\text{BH}} = \frac{A}{\alpha}$