Testing General Relativity with gravitational waves in the presence of matter effects within Neutron Star mergers

Reed Essick
MIT LIGO Lab

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Black Hole Initiative
Consider Compact Binary Coalescences (CBCs) with at least one NS
→ NS spins are (assumed) small
→ BH spins may not be small or aligned
Test Infrastructure for General Relativity

Bayesian model selection

\[
O^A_B = \frac{p(A|\text{data})}{p(B|\text{data})}
\]
Test Infrastructure for GEneral Relativity

Bayesian model selection

Odds Ratio

\[ O_A^B = \frac{p(A|\text{data})}{p(B|\text{data})} \]

\[ = \frac{p(\text{data}|A)p(A)}{p(\text{data}|B)p(B)} \]
**Test Infrastructure for General Relativity**

Bayesian *model selection*

\[ \text{Odds Ratio} \quad \frac{O^A_B}{\text{ratio of evidences}} = \frac{p(A|\text{data})}{p(B|\text{data})} = \frac{p(\text{data}|A)p(A)}{p(\text{data}|B)p(B)} \]

\[ = \left( \frac{\int d\theta p(\text{data}|\theta, A)p(\theta|A)}{\int d\varphi p(\text{data}|\varphi, B)p(\varphi|B)} \right) \left( \frac{p(A)}{p(B)} \right) \]

\[ \text{prior odds} \]
Test Infrastructure for General Relativity

Bayesian model selection

Odds Ratio

\[ O_{AB}^A = \frac{p(A|\text{data})}{p(B|\text{data})} = \frac{p(\text{data}|A)p(A)}{p(\text{data}|B)p(B)} = \left( \frac{\int d\theta p(\text{data}|\theta, A)p(\theta|A)}{\int d\vartheta p(\text{data}|\vartheta, B)p(\vartheta|B)} \right) \left( \frac{p(A)}{p(B)} \right) \]

ratio of evidences

prior odds

Evidence

\[ Z_A = \int d\theta \frac{p(\text{data}|\theta, A)p(\theta|A)}{p(\theta|A)} \]

likelihood

prior

(known for stationary Gaussian noise)
Test Infrastructure for GEneral Relativity

Bayesian model selection

\[ O^{!PP}_{PP} = \]

point-particles+deviations (\(!PP\)) \hspace{1cm} 15+N model parameters

\hspace{1cm}

point-particles (PP) \hspace{1cm} 15 PP parameters

\hspace{1cm}

N parametrized deviations

\hspace{1cm}

15 model parameters
intrinsic : Masses, spins
extrinsic : location, inclination
Test Infrastructure for GEneral Relativity

Bayesian model selection

\[
O^{!PP}_{PP} = \frac{\text{point-particles+deviations} \quad \text{15+N model parameters}}{\text{point-particles} \quad \text{15 PP parameters}}
\]

\[
\text{intrinsic : Masses, spins} \quad N \text{ parametrized deviations}
\]

\[
\text{extrinsic : location, inclination}
\]

Modified GR model introduces changes to PN expression for GW phase

\[
\Psi = 2\pi ft_c - \varphi_c - \frac{\pi}{4} + \sum \left[ \psi_j + \psi_j^{(l)} \ln f \right] f^{(j-5)/3}
\]

\[
\psi_j \rightarrow (1 + \delta \chi_j)\psi_j^{GR}
\]
Linear Tides and Merger

High frequency signal is *messy*
Linear Tides and Merger

High frequency signal is *messy*

**Solution:**
Truncate likelihood well *within the inspiral*
Linear Tides and Merger

High frequency signal is *messy*

**Problem:** Nonlinear tides affect the inspiral at *low frequencies*

**Solution:** Truncate likelihood well *within the inspiral*
Nonlinear Tides

Wave-wave interactions can be important even for small displacements if the coupling coefficient is large.

Daughter modes begin to grow exponentially when the orbital frequency exceeds \(~50\) Hz.

Tide-gp coupling is important and instability is non-resonant.

\[
\epsilon = \frac{\delta R}{R} \sim \frac{m}{M} \left( \frac{R}{a} \right)^3
\]
Nonlinear Tides
dissipation through $g$-mode breaking

$$\dot{E} \approx \Gamma N E_{\text{sat}} \sim A \left( \frac{f}{f_{\text{ref}}} \right)^n \Theta(f - f_0)$$
Nonlinear Tides
dissipation through \textit{g-mode breaking}
Nonlinear Tides

dissipation through \textit{g-mode breaking}

\[ \dot{E} \approx \Gamma N E_{\text{sat}} \sim A \left( \frac{f}{f_{\text{ref}}} \right)^n \Theta(f - f_0) \]

dissipation rate depends the energy at which modes saturate, how many modes participate, and the exact growth rate

saturation’s evolution with frequency is uncertain, but is likely a combination of the orbital separation and the \textit{p-, g-mode} spectra
Nonlinear Tides
dissipation through $g$-mode breaking

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dissipation rate depends the energy at which modes saturate, how many modes participate, and the exact growth rate

saturation's evolution with frequency is uncertain, but is likely a combination of the orbital separation and the $p$, $g$-mode spectra

Growth rates are large compared to the GW inspiral timescale at low frequencies, but turn-on frequency is uncertain.
Nonlinear Tides impact on GW waveform

\[ \dot{E} \approx \Gamma N E_{sat} \sim A \left( \frac{f}{f_{ref}} \right)^n \Theta(f - f_0) \]

For a (1.4Msun, 12km) NS:

- \( A \) between \( 10^{-10} - 10^{-5} \)
- \( n \) between 0 - 2
- \( f_0 \) between 10 Hz – 100 Hz

\( A \sim 4 \times 10^{-6} \)
Nonlinear Tides

impact on GW waveform

energy balance

$$\frac{d}{dt}E_{\text{orb}} = -\mathcal{L}_{GW} - \dot{E}_1 - \dot{E}_2$$

produces an additional phase correction

$$\Delta \phi \propto M^{-10/3} \left[ \left( \frac{m_1}{M} \right)^{2/3} \frac{A_1}{n_1 - 3} \left( \left( \frac{f_1}{f_{\text{ref}}} \right)^{n_1 - 3} - \left( \frac{f}{f_{\text{ref}}} \right)^{n_1 - 3} \right) + (1 \leftrightarrow 2) \right]$$

and we expand each tidal parameter as a function of the component mass

$$A_i = A_i^{(0)} + A_i^{(1)}(m_i - 1.4 M_\odot) + \cdots$$
Nonlinear Tides

What happens when we

1. inject NL phase shift
2. recover with PP model

As the amplitude increases, we lose evidence for the PP model.

We will only see a fraction of signals

\[ f_{\text{obs}} = \left( \frac{\rho_{\text{rec}}}{\rho} \right)^3 \]

Essick et al. (2016)
Nonlinear Tides impact on GW waveform

What happens when we
(1) inject NL phase shift
(2) recover with PP model

Essick et al (2016)

SNR~25

What happens when we
(1) inject NL phase shift
(2) recover with NL model

Essick et al (2016)
Nonlinear Tides impact on GW waveform

Essick et al (2016)

\[ A = 1.6 \times 10^{-7} \]

\[ n = 2 \]

\[ f_0 = 50 \text{ Hz} \]

\[ \ln O_{PP}^{NL} = 53 \]
Nonlinear Tides impact on GW waveform

Essick et al (2016)

\[ \ln O_{PP}^{NNL} = 53 \]

\[ \ln O_{PP}^{NP} = 45 \]

\( n = 2 \)

\( f_0 = 50 \text{ Hz} \)

\[ A = 1.6 \times 10^{-7} \]
References

**TIGER: A data analysis pipeline for testing the strong-field dynamics of general relativity with gravitational wave signals from coalescing compact binaries.**
M. Agathos et al. PRD 89, 082001 (2014).

**Matter effects on binary neutron star waveforms.**
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**Impact of the tidal p-g instability on the gravitational wave signal from coalescing binary neutron stars.**

**Growth rate of the tidal p-mode g-mode instability in coalescing binary neutron stars.**

**The stability of tidally deformed neutron stars to three- and four-mode coupling.**

**An instability due to the nonlinear coupling of p-modes to g-modes: implications for coalescing neutron star binaries.**
NL Tides

Essick et al (2016)
NL Tides

Essick et al (2016)

SNR~25
n=0
f₀=50 Hz

\[ p(M|\text{data}) \]

\[ q \]

\[ \mathcal{M}/M_\odot \]

\[ p(q|\text{data}) \]

\[ 2.5 \times 10^{-8} \]

\[ 6.3 \times 10^{-8} \]

\[ 1.6 \times 10^{-7} \]
Essick et al (2016)

$SNR \sim 50$

$n = 2$

$f_0 = 50 \text{ Hz}$
Essick et al (2016)

SNR~25
n=0
f_0=50 Hz

A

1.6 \times 10^{-7}
6.3 \times 10^{-8}
2.5 \times 10^{-8}
1.0 \times 10^{-8}
A = 0

p(M|data)

10^{-9} 1.2184 1.2186 1.2188 1.2190

\mathcal{M}/\mathcal{M}_\odot

10^{-3} 10^{-2} 10^{-1} 10^0

p(A|data)
NL Tides

Essick et al (2016)

SNR~25

$n=0$

$f_0=50$ Hz
SNR~12

\( n=0 \)

\( f_0 = 30 \text{ Hz} \)
NL Tides

$\text{SNR} \approx 12$

$n=0$

$f_0 = 50 \text{ Hz}$
NL Tides

\[ \text{SNR} \approx 25 \]

\[ n = 0 \]

\[ f_0 = 80 \text{ Hz} \]