Singularity Resolution Through Dynamical Quantum Gravity

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First Annual BHI Conference on Black Holes
Black Hole Initiative
Harvard University
09 May 2017
Singularity resolution through dynamical quantum gravity

- My take home message of yesterday:  
  **GR black holes probably exist!**
- Singularities are ubiquitous in General Relativity (e.g., singularity theorems).
- Hope: quantum effects could ease or remove (some/all) singularities.
- Uncertainty principle could ‘smooth out’ pathologies.
- Dashed hope: naive ‘timeless’ (Wheeler–DeWitt) quantization fails to resolve even simple cosmological singularities.

What goes wrong?

⇒ We believe it is the treatment of time
Singularity resolution through dynamical quantum gravity

- I will give a quantization of a simple cosmological model that:
  - has genuine evolution, and
  - resolves the singularity that persists in the timeless approach.
- In addition, the model exhibits novel phenomenology including:
  - an inflationary epoch,
  - a macro parameter encoding deep quantum physics, and
  - a new platform for condensed matter quantum simulation of the early universe.
- Could be applied to more general kinds of singularities.
The classical model

A simple FRWL cosmology

Homogeneous and isotropic mini-superspace model with massless scalar field and vanishing spatial curvature.

Hamiltonian:

\[ H = N \left[ -\frac{\kappa}{12V_0a}\pi_a^2 + \frac{1}{2V_0a^3}\pi_\phi^2 + \frac{V_0a^3}{\kappa}\Lambda \right] \]

Variables

- \( a \) - scale factor
- \( \phi \) - scalar field

Parameters

- \( \Lambda \) - cosmological constant
- \( \pi_\phi \) - momentum of scalar field

\((V_0 \text{ - fudicial volume; } \kappa \text{ - Newton constant})\)
The Hamiltonian can be written as

$$ H = N \left( g^{ab} p_a p_b - \frac{\Lambda}{2} \right) $$

(1)

$g^{ab}$ ⇒ inverse Minkowski metric on configuration space $C$!

Solutions ⇒ geodesics on $(C, g)$

**Restriction**

Geometrically, $a > 0$ puts us in Rindler spacetime.

Boundary ⇒ geodesic incompleteness on $(C, g)$ (classical singularity and quantum complications)
Boost symmetry suggests Rindler coordinates: $v \propto a^3$, $\varphi \propto \phi$

$$v = v_0 + \sqrt{t - 1}$$
$$\varphi = \varphi_\infty + \text{arctanh} \left( t^{-1} \right)$$

### Parameters
- $v_0$ - initial time
- $\varphi_\infty$ - asymptotic $\varphi$
- $\Lambda$ - total energy
- $\pi_\varphi$ - conserved $p$

### Symmetries
- Time translations
- Boost invariance
- Time units
- Space units

Symmetries $\Rightarrow$ no physically relevant parameters!
Quantization Preliminaries

Hamilton–Jacobi equation:

\[ \mathcal{H} \left( q, \frac{\partial S}{\partial p} \right) = \frac{\Lambda}{2} \]

Quantization ambiguity

\[ \hat{\mathcal{H}} \psi = \frac{\Lambda}{2} \psi \quad \text{or} \quad \hat{\mathcal{H}} \psi = i \frac{\partial \psi}{\partial \tau} \]

⇒ Timeless (Wheeler–DeWitt) vs genuine evolution (unimodular gravity-like)

Relational quantization:

- Take approach with genuine evolution.
- \( \tau \) - unobservable parameter ordering successive states.
Wheeler–DeWitt equation is timeless.

⇒ deparametrize wrt internal time. (e.g., $\varphi$)

**Problem (example)**

$<v>$ problematic when $\varphi \to \infty$ (takes definite values)

Relation quantization $\Rightarrow \hat{\varphi}$ does not take definite values.

$<v>, <\varphi>$ protected by uncertainty relations!

We will use genuine evolution:

$$\hat{\mathcal{H}} \psi = \Box \psi = i \frac{\partial \psi}{\partial \tau}.$$
Unitarity

Boundary of $\mathcal{C} \Rightarrow \hat{\mathcal{H}}$ not essentially self-adjoint!

$$\langle \Phi, \hat{\mathcal{H}} \Psi \rangle = \langle \hat{\mathcal{H}} \Phi, \Psi \rangle + \text{boundary}$$

(Boundary term $\Rightarrow$ quantum trace of classical singularity.)

**Self-adjoint Extensions**

- Guaranteed by von Neumann. ($\hat{\mathcal{H}}$ - symmetric and real)
- Trick: the equation (boundary $= 0$) is conformally invariant.
- Use eigenfunctions on conformal completion $\tilde{g} = \nu^2 g$ (maps $\mathbb{R}^+ \times \mathbb{R} \rightarrow \mathbb{R}^2$) to anchor solutions in Minkowski.
- Put phase between solutions to remove $\Lambda$-(scale) dependence.

$\Rightarrow$ must introduce reference scale $\Lambda_{\text{ref}}$. 
‘Bound’ states ($\Lambda < 0$)

Spectrum of Klein–Gordon operator on Rindler for $\Lambda < 0$:

- AdS-like states exit.
- Spectrum is discrete but unbounded: ($k$ - momentum of $\varphi$)

$$\Lambda_n = \Lambda_{\text{ref}} e^{2\pi n/k} \quad (n \in \mathbb{Z})$$

- Conformal tower of states with accumulation point at $\Lambda = 0$.
- Analogues of atomic systems: *Efimov effect*.
- Eigenstates behave like $\psi_{\Lambda} \sim e^{-\nu}$.

$\Rightarrow$ no late-time semi-classical limit!
dS-like states exist and persist to late-time, semi-classical limit.

Continuous spectrum given by phase-shifted Bessel functions:

\[ \psi_\Lambda \sim J_{ik}(\sqrt{\Lambda}v) + \tan \theta I_{ik}(\sqrt{\Lambda}v) \]

\[ \theta \propto k \log \Lambda/\Lambda_{\text{ref}} \quad (U(1) \text{ s.a. ext. parameter}). \]

Gaussian states lead to semi-classical physics.

New dimensionful parameter \( \Lambda_{\text{ref}} \) gives meaning to size of quantum effects (\( \hbar \)).

Self-adjoint extension parameter: phase between in- and out-state.
Analogue model

- System is mathematically equivalent to $N$-particle system with $1/r^2$ potential (in a particular regime).
- Regime in question is a good effective model for some 3-body atomic systems.
- Known as *Efimov effect* $\Rightarrow$ vast literature.
- Unbound states can be scattered off bound states.
- Self-adjoint extension parameter gives scattering length via phase shift, $\theta$.
- $\theta$ is macro parameter encoding micro-physics.

How seriously can we take the analogue model or simulator of the early universe?
Bounce Solution
Expectation values

\[ <v> \] has a min when \( v_{\text{class}} = 0 \).

\[ <\phi> \] is finite when \( \phi_{\text{class}} \to \infty \).

Singularity resolved and classical limit recovered!
New parameter, \( \Lambda_{\text{ref}} \), gives meaning to \( \hbar \) and relative size of quantum effects.
Simple example in $\hbar \sim 1$ regime.

Note inflationary period where $\frac{\partial \langle \phi \rangle}{\partial \langle v \rangle} \approx 0$. 
We have given a quantization of a simple FRLW cosmology that

- Resolves singularity (improvement over WDW).
- Recovers semi-classical dS limit.
- Suggest novel phenomenology in quantum regime.
- Provides unique macro-parameter, $\Lambda_{\text{ref}}$, encoding micro-physics.
- New platform for condensed matter quantum simulation of the early universe.
Generalizations

- Bianchi I: $\nu$ wavefunction is identical.
- Bianchi IX: Bessel functions $\Rightarrow$ non-analytic with same asymptotic properties.
- BKL $\Rightarrow$ more general mechanism for singularity resolution via quantum effects.
- Conformal anomaly and holographic renormalization.
- Absorption/Decay: AdS-‘bound’ state $\Rightarrow$ dS-‘emission’ (need non-linear corrections).
- Self-adjoint extension parameter $\Rightarrow$ UV-completion required (conformal?).