Listening to the Chirps: Estimating black hole parameters using the LIGO results
Thank you to Avi Loeb, Peter Galison, Erik Curiel, and the Black Hole Initiative.

Thoughts to Hanford, Washington, where a tunnel has just collapsed near the Plutonium Uranium Extraction Facility.
Discovery, Justification, and Pursuit

Hans Reichenbach, *Experience and Prediction*:

**Context of Discovery.** The messy process of actually discovering results in science. Anarchic, contingent, and irritating.

**Context of Justification.** The process of 'rationally reconstructing' and writing up one's results. Presents the process of discovery as smooth, logical, and quick.

Larry Laudan, *Progress and its Problems*, and Allan Franklin, *Fifth Force*:

**Context of Pursuit.** A theory or idea is considered worth pursuing by the scientific community. Focusing on pursuit allows us to study heuristics: what methods and techniques allow for hypothesis testing, expansion of theories, model construction.
Heuristic Models

Focusing on experimental science and its role in theory building has a real payoff - you learn about:

1. How models are generated from data, and theories from testing.
2. Which elements of a theory or model depend on which.
3. How each element of a theory or model can be regarded as part of a class of such elements, and what the principle of membership in the class is.

For instance, a Kerr black hole is a member of a class of black holes that also includes Schwartzschild black holes, which have no charge or momentum. Which kind of black hole a given BH is depends on properties like mass, charge, and spin. This is elementary but important.
Every detection encodes information.

Not just about the target system, but also about the detector, the model of the detector, and the possible ways of generating models of the data and of the systems under investigation.

The data allows for estimation of the system parameters.

In a recent paper, "Tests of GR using GW150914", the LIGO researchers propose ways of estimating parameters of the binary black hole (BBH) system using the data.

The LIGO sessions in this conference have mentioned others.
Tests of GR with LIGO

Emanuele Berti and Deirdre Shoemaker: Is the black hole a Kerr mass or not?

Sheperd Doeleman (et al), Dan Holz, Scott Hughes, Matthew Evans: Improved detectors will allow for cross-confirmation and:

(a) More comprehensive scanning (Doeleman et al)
(b) Better localization (Holz)
(c) Direct observation (Doeleman)
Strong gravity and beyond Standard Model physics

Modification
- Modified GR (Ultraviolet/Infrared)
  Most theories: same BHs as GR
  Dynamics can be different

\[ G_{\mu\nu} \]

Can we test...
- Kerr dynamics?
  No-hair tests with ringdown
  Modified gravity: Horndeski, EdGB...

- Quantum corrections at the horizon?
  Firewalls
  Echoes

- Beyond Standard Model physics
  \[ T_{\mu\nu} \]

- Ultralight dark matter candidates?
  Axions; superradiant instabilities

- Exotic compact objects & beyond SM?
  Rule out exotica
  (boson stars, Proca stars, gravastars...)

Emanuele Berti: Testing the Kerr Paradigm with Gravitational Wave Observations
Signal processing and parameter estimation

I propose to focus on the path from the data to the waveform, which has a truly amazing history, which I can't cover here.

And then from the waveform to estimating parameters, and from there to theory testing.
Black hole merger

We begin with interferometer data. A billion light years away, two black holes spiral around each other briefly and then merge to form a single, more massive black hole. Gravitational waves are ripples in spacetime that propagate from this event; they have a tiny wave strain ($1.0 \times 10^{-21}$) that can nonetheless be picked up by an interferometer.

There are two interferometers: one at Livingston, one at Hanford.
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LIGO: parameter estimation

If we can infer from the LIGO data that the orbital period of the observed Binary Black Hole (BBH) system is not constant, that is a test of general relativity. In "Tests of General Relativity with GW150914," LIGO researchers write,

In Newtonian gravity, binary systems move along circular or elliptical orbits with constant orbital period [4, 5]. In GR, binary systems emit GWs [6, 7]; as a consequence, the binary’s orbital period decreases over time as energy and angular momentum are radiated away. Electro-magnetic observations of binary pulsars over the four decades since their discovery [8, 9] have made it possible to measure GW-induced orbital-period variations $\dot{P}_{\text{orb}} \sim -10^{-14} - 10^{-12}$, confirming the GW luminosity predicted at leading order in post-Newtonian (PN) theory [10] (i.e., Einstein’s quadrupole formula) with exquisite precision [11, 12].
The gravitational waveform $h(t)$ depends on the chirp mass of the binary, $\mathcal{M} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$ [32, 33], the symmetric mass ratio $\eta = (m_1 m_2) / (m_1 + m_2)^2$ [34], and the angular momentum of the compact objects $\chi_{1,2} = cS_{1,2} / Gm_{1,2}^2$ [35, 36] (the compact object’s dimensionless spin), where $S_{1,2}$ is the angular momentum of the compact objects. The effect of spin on the waveform depends also on the ratio between the component objects’ masses [37]. Parameters which affect the overall amplitude and phase of the signal as observed in the detector are maximized over in the matched-filter search, but can be recovered through full parameter estimation analysis [18]. The search parameter space is therefore defined by the limits placed on the compact objects’ masses and spins. The minimum component masses of

**Chirp mass**: the combination of the total mass and the reduced mass as energy is radiated away, which "determines how fast the binary sweeps, or chirps, through a frequency band" (https://www.astro.umd.edu/~miller/teaching/astr498/lecture25.pdf)
Signal Processing

Eric Chassande-Mottin, Patrick Flandrin, Archana Pai, and others have made contributions to the study of chirping systems, and in particular to detection and to signal processing.

Alessandra Buonanno and others have investigated the implications for future data analysis and mathematical modeling.
On the Time–Frequency Detection of Chirps

Eric Chassande-Mottin and Patrick Flandrin

As far as detection itself is concerned (from a signal processing point of view), a key question is to get some \textit{a priori} information about possible structures for the expected waveforms. In fact, a wide variety of situations can be considered [32], each corresponding to different types of signals, more or less well characterized. Nevertheless, it is
**FIG. 1.** Qualitative validity of the chirp interpretation for gravitational waves. Gravitational waves radiated by coalescing binaries can be considered as chirps as long as their maximum frequency is much smaller than a critical frequency which depends on the masses $m_1$ and $m_2$ of the binary. This diagram plots (solid lines) this critical frequency when $m_1$ varies between $1 M_\odot$ and $10 M_\odot$, and when $m_2 = k m_1$, with $1 \leq k \leq 10$. The dotted line (plotted here arbitrarily at 500 Hz) stands for the high-frequency cutoff of the detector, which allows one to get a rough bound for the validity of the approximation.

**FIG. 5.** Detection of a gravitational wave. This figure illustrates the efficiency of an optimum time-frequency-based detection for a gravitational wave with coalescence time $t = 0$ of a binary composed of two objects of $1 M_\odot$ and $10 M_\odot$ at a distance of 200 Mpc in case (a) and 1 Gpc in case (b). Since the distance between the binary and Earth changes the only signal amplitude, the signal-to-noise ratio is the only parameter that has been modified between these two examples. Each plot compares the squared envelope of the output of the matched filter (dashed-dotted line) with a time-frequency strategy based on a line integration over either a classical spectrogram (dashed line) or its reassigned version (solid line). In order to make appear more clearly what is gained in terms of contrast, the maximum of each of these curves has been arbitrarily normalized to unity.
In the examples of Fig. 5, the chirp mass $M_\odot$ was implicitly assumed to be known, which is by no means the case in practice. Assuming that $M_\odot$ is unknown, a refined strategy amounts to applying the previous one in parallel by performing as many line integrations as is necessary for sampling values of $M_\odot$ over some expected range. Figures 6 and 7 exhibit the application of this strategy on the reassigned spectrogram and standard spectrogram, respectively. This joint detection–estimation problem allows also for an estimate of $M_\odot$ to be obtained. It should be noted that, when scanning test values for $M_\odot$, the reference signal energy is modified. The output of each detector therefore must be divided by a factor proportional to the squared amplitude of the reference signal (which varies as $M_\odot^{5/3}$) in order to compare coherent results.
A chirplet is a short piece of signal whose frequency varies linearly between two successive nodes of the grid. In modulated signal i.e., a chirp. In this article, we address the question of detecting an a priori unknown GW chirp. We introduce a general chirp model and claim that it includes all physically realistic GW chirps. We produce a finite grid of template waveforms which samples the resulting set of possible chirps. If we follow the classical approach (used for the detection of inspiralling binary chirps, for instance), we would build a bank of quadrature matched filters comparing the data to each of the templates of this grid. The detection would then be achieved by thresholding the output, the maximum giving the individual which best fits the data. In the present case, this exhaustive search is not tractable because of the very large number of templates in the grid. We show that the exhaustive search can be reformulated (using approximations) as a pattern search in the time-frequency plane. This motivates an approximate but feasible alternative solution which is clearly linked to the optimal one. The time-frequency representation
Matched filtering

*with detector modeling and chirp classification*

This is a method of matched filtering, but note that it is based on

1. A model of how different chirping systems can be detected given detector sensitivity, and

2. A classification of chirping systems and exactly how they are modeled, using best chirplet chains or other methods.
Models of system dynamics

A logical next step would be to begin with the template bank mentioned in a previous slide, and then to build a family of models of binary black hole (and neutron star) system dynamics based on physically realistic values of the parameters.

Such a family of models would provide an a priori standard for detections and for evaluating candidate waveforms. (And may already be happening.)
Models and theory development

If you don't focus on the experimental work being done and on the development of theoretical and formal tools as embedded in that experimental work, you miss a great deal.

Focusing only on what philosophers call the context of justification misses all of the material in the context of pursuit - the way that a scientific classification is set up in the first place, for instance (in this case, "chirping systems").

This material is crucial for contemporary scientific theories about realism (that scientific theories describe reality as it is), among others.
Heuristics and increased understanding

The work cited here provides an a priori framework for the detection and modeling of the data that at the same time classifies that data within a family of chirping system. At the same time, it makes more explicit the relationships between data, model, and theory.

From a philosophical perspective, this is a step forward. Generally, the LIGO results (and related results) provide unparalleled opportunities to study how our understanding of dynamical systems, of families of phenomena, and of scientific modeling is built and extended.
The Future

The LIGO experiments are the tip of the iceberg. LISA, VIRGO, KARGO, and the Event Horizon Telescope are undertaking more and more ambitious experiments, and LIGO itself will ramp up its efforts.

The experiments provide data that can be the basis of deep investigations of general relativity, of our understanding of black holes, quasars, neutron stars, and their interactions.

One way for philosophers to work in a way that is complementary to this scientific research is to focus on how we can learn from the data and how it is modeled, and how the dynamics of experimental research informs theory building.
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